

# Considerations of A Two-Fluid Heliospheric Plasma Dynamics Under Dominant Electron Pressure

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**Abstract**

Electrons in plasma physics mostly are the underestimated species, since usually they only have to guarantee electric quasineutrality, but don't count in terms of mass-, momentum-, and energy flows. This is different in space plasmas like the heliospheric plasma, especially the plasma downstream of the solar wind termination shock. Here it has become evident more recently that electrons dominate the plasma pressure and, connected with that, the plasma energy flow. Under these conditions a two-fluid plasma theory is needed to adequately describe fields and flows. We first here develop a pure two-fluid thermodynamics of such two-fluid plasmas and then study the actual situation in case of the heliospheric plasma that the electron pressure is dominating over the proton pressure. Under such auspices the electron pressure determines the mass- and momentum flows of the plasma and in fact decreases with the decrease of bulk velocity of the flow.

**Introduction**

One may imagine a plasma consisting of heavy protons and light electrons, however, the protons are dominating the mass- and momentum- flows, while the electrons dominate the energy flow, i.e. a "hot electron fluid" in a massive background of cold protons! Then under these auspices the change of the electron energy flow at the actual plasma motion causes the changes both of the mass- and of the momentum- flows of the protons. Such a twin-fluid dynamical situation, for example, in fact occurs just downstream of the solar wind termination shock where electrons get shock-accelerated due to the action of the shock-electric field and thereafter get thermally randomized into a hot and pressurized electron fluid [1-3].

Another situation leading to similar two-stream plasma phenomena comes up in the solar wind near the solar corona where electrons and protons generate different temperatures [4-7]. While, however, for the solar wind case the effective energy sources for electrons and protons essentially are open guesses, in the heliosphere downstream of the solar wind termination shock, the actually given facts are much better defined. Therefore, we here shall take a look on the two-fluid properties of the plasma flow downstream of the termination shock.

**Study of the Two-Fluid Situation in the inner Heliosphere**

For a better clarification of this special point of a two-fluid plasma situation in the inner heliosphere we look here at the given conditions from a slightly different view, namely following first the

standard thermodynamical procedures, stating that the work done by the pressure at a change of the co-moving plasma volume  $\Delta W$  is reflected by an associated change of the internal energy  $\epsilon$  of that volume, if no other non-LTE effects are operating, like conservation of magnetic moments or wave-particle interactions or heating by compressive MHD waves, which, however, will be taken into account later on here. First then the action of the pressures requires that in the Solar Rest Frame (SRF) the following rarely used equation has to be respected:

$$-(P_e + P_i) \frac{d\Delta W}{ds} = \frac{d}{ds} [(\epsilon_i + \epsilon_e)\Delta W]$$

where  $s$  is the streamline coordinate, and  $\Delta W$ , as explained in Fahr and Dutta-Roy, [8] denotes the co-moving plasma volume on the streamline, i.e. a fluid volume that locally co-moves with the common plasma bulk velocity of electrons and protons,  $\vec{V} = \vec{V}_i = \vec{V}_e$ . Hereby the indices "i, e" indicate ion- or electron- related quantities - as e.g. densities  $n_{i,e}$ , pressures  $P_{i,e}$  and internal energies  $\epsilon_{i,e}$ , respectively [8]. We furthermore do consider here the dynamic plasma structure on spatial resolution scales large in comparison to scales of the order of Debye lengths  $\lambda_D$ , where space charges and electric currents play a role. We instead require here in our further study equal densities  $n = n_e = n_i$  and bulk velocities  $\vec{V} = \vec{V}_i = \vec{V}_e = M$

Then one must take into account the fact that in the SRF the ion energy density is given by  $\epsilon_i = nMV^2/2 + (3/2\pi)P_i$ , while the electron energy density only is given by  $\epsilon_e = (3/2\pi)P_e$  (i.e.: strongly

subsonic electron flow:  $V \ll c_e$  !;  $c_e$  denoting the mean thermal electron velocity). When furthermore assuming, in order to start the business from some concrete basis, that the electron pressure competes with or even dominates over the ion pressure, i.e.  $P_e \geq P_i$ , (i.e. in fact the case given just downstream of the heliospheric termination shock, will then bring the upper relation to the following net equation [2, 9-12]:

$$-(P_e + P_i) \frac{d\Delta W}{ds} = \frac{d}{ds} \left[ (nMV^2/2 + \frac{3}{2\pi}(P_e + P_i)) \cdot \Delta W \right]$$

When additionally recognizing here that for an incompressible flow, as given here in case of the strongly subsonic flow with  $c_e \gg V$ , the comoving plasma volume is given simply by the following relation  $\Delta W = \Delta W_0 \cdot (V_0/V)$  [8]. Then the above equation simplifies into the form:

$$-(P_e + P_i) \frac{d\frac{1}{V}}{ds} = \frac{d}{ds} \left[ \frac{1}{2}nMV + \frac{3}{2\pi} \frac{d}{ds} ((P_e + P_i) \frac{1}{V}) \right]$$

which for  $n = \text{const}$ ;  $dn/ds = 0$  leads to

$$-(P_e + P_i) \frac{d\frac{1}{V}}{ds} = \frac{1}{2}nM \frac{d}{ds} V + \frac{3}{2\pi} (P_e + P_i) \frac{d}{ds} \frac{1}{V} + \frac{3}{2\pi} \frac{1}{V} \frac{d(P_e + P_i)}{ds}$$

and further simplifies to:

$$\left[ \frac{2\pi+3}{2\pi} (P_e + P_i) - \frac{1}{2}nMV^2 \right] \frac{d}{ds} V = \frac{3}{2\pi} V \frac{d}{ds} (P_e + P_i)$$

In view of this above relation, let us now consider the special situation of a plasma with dominant electron pressure  $P_e$  i.e.  $P_e$  strongly dominating over the ion pressure  $P_i$ . Then the upper equation first reduces to:

$$\left[ \frac{2\pi+3}{2\pi} P_e - \frac{1}{2}nMV^2 \right] \frac{dV}{ds} = \frac{3}{2\pi} V \frac{dP_e}{ds}$$

This equation seems to require a solution of the form  $P_e = P_e(V)$ , and with  $dV/ds \geq 0$  (exclusion of the heliopause stagnation point) leads to

$$\frac{2\pi+3}{2\pi} P_e - \frac{3}{2\pi} V \frac{dP_e}{dV} = \frac{1}{2}nMV^2$$

Obviously the solution of the upper differential equation requires  $P_e$  to be a function of  $V$ , tentatively by the following representation

$$P = P_{e0} \cdot (V/V_0)^\delta + C_e$$

which in connection with the upper differential equation leads to the request:

$$P_e(V) = \frac{\pi}{2\pi+3-3\delta} nMV^2$$

which all together with  $\delta=2$  then yields the relation:

$$P_e(V) = \frac{\pi}{2\pi+3-3\delta} nMV^2 = \frac{\pi}{2\pi-3} nMV^2$$

To come to an idea of what actual pressure should be used for  $P_{e0}$ , one could try to calculate the initial downstream electron pressure for that above relation by using the context that for reasons of the entropy generation at the termination shock the downstream pressure is connected with the loss of kinetic energy of the plasma flow between upstream (index "1") and downstream (index "2") [1]:

$$\Delta P_{1,2} = P_{e0} \cdot (V_1/V_0)^2 - P_{e0} \cdot (V_2/V_0)^2$$

and with  $V_0 = V_2$  as the downstream bulk velocity one would obtain:

$$\Delta P_{1,2} = P_{e0} \cdot (V_1/V_2)^2 - P_{e0} \cdot (V_2/V_2)^2 = P_{e0} \cdot [s^2 - 1]$$

where 6 denotes the shock compression ratio (according to VOYAGER-2 measurements with  $6 = 2.5$ ) [13]:

$$P_{e0} = \frac{\Delta P_{1,2}}{[6^2 - 1]}$$

When assuming that at the solar wind termination shock the loss of kinetic energy of the plasma flow is converted into pressure energy of the electrons, then we obtain:

$$\Delta P_{1,2} = \frac{1}{2} M \cdot [n_1 V_1^2 - n_2 V_2^2] = (1 - \frac{1}{6}) \left[ \frac{1}{2} M n_1 V_1^2 \right]_{VOY-2}$$

where the indices "1" and "2" indicate upstream and downstream solar wind quantities left and right of the termination shock, and the quantity  $(1 - 1/6)$  for  $6 = 2.5$  evaluates to 0.6. Hence one obtains the following explicit number:

$$P_{e0} = \frac{\Delta P_{1,2}}{[s^2 - 1]} = \frac{(1 - \frac{1}{6}) \left[ \frac{1}{2} M n_1 V_1^2 \right]_{VOY-2}}{[6^2 - 1]} = \frac{1}{6(6+1)} \left[ \frac{1}{2} M n_1 V_1^2 \right]_{VOY-2} = 0.114 \left[ \frac{1}{2} M n_1 V_1^2 \right]_{VOY-2}$$

When inserting all of this into the upper differential equation, one then finds the requirement

$$\frac{2\pi-3}{2\pi} (P_{e0} \frac{V^2}{V_0^2} + C) = \frac{1}{2} nMV^2$$

or yielding the initial electron pressure downstream of the shock in the form:

$$P_{e0} + C = \frac{2\pi}{2\pi-3} \frac{1}{2} nMV_0^2 = 0.96 \cdot n_1 MV_0^2$$

With that result, coming now back to the fact that the electron pressure performs thermodynamical work, when pumping down the streamline the electron-proton plasma, one must conclude that without any interaction of ions and electrons, this energy, which has to be thermodynamically expended, has to be taken from the internal thermal energy  $\epsilon_e$  of the electrons themselves. This leads to the following term describing the decrease of the electron thermal energy

$$-P_e \frac{d\Delta W}{ds} = \frac{d}{ds} [\epsilon_e \Delta W] = \frac{d}{ds} \left[ \frac{3}{2\pi} P_e \Delta W \right]$$

Together with the relation for the comoving fluid volume in incompressible flows  $\Delta W = (W_0) \cdot (V_0/V)$  this consequently leads to the following expression [8]:

$$-P_e \frac{d}{ds} \frac{1}{V} = \frac{d}{ds} \left[ \frac{3}{2\pi} P_e \frac{1}{V} \right] = \frac{3}{2\pi} \left[ P_e \frac{d}{ds} \frac{1}{V} + \frac{1}{V} \frac{dP_e}{ds} \right]$$

which can be simplified to

$$-\frac{2\pi}{3} \left( 1 + \frac{3}{2\pi} \right) V \frac{d}{ds} \frac{1}{V} = \frac{1}{P_e} \frac{dP_e}{ds}$$

and consequently yielding a pressure change due to volume work as given by:

$$\frac{d \ln P_e}{ds} = \frac{2\pi + 3}{3} \frac{d \ln V}{ds}$$

This would require the electron pressure  $P_e$  to be a function  $P_e = P_{e0} \cdot (V/V_0)^2 + C_e$  of the bulk velocity  $V$  as already found further above.

### A more general theoretical approach

This final relation of the above section, however, only correctly describes the puristic case that solely the fluid thermodynamics determines the electron pressure  $P_e$ . When this, to better meet realistic conditions, is combined with other influencing terms in a more general pressure transport equation (e.g. see Equ. (14) [8], in then it leads to the following more complete form of a pressure transport equation written as a differential equation with respect to the line element  $s$  along the streamline [8]:

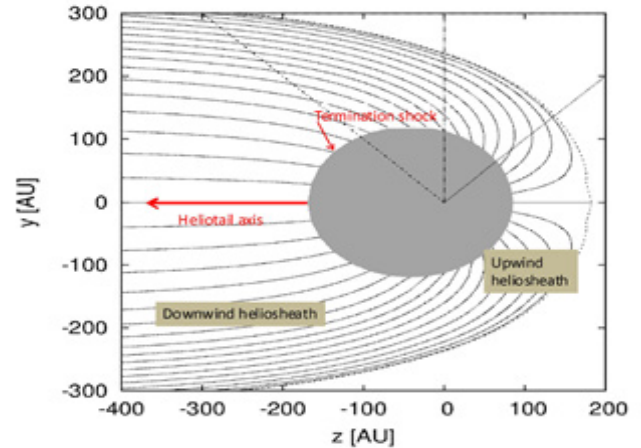
$$\frac{d \ln P_e}{ds} = \frac{4}{3} \frac{d \ln B}{ds} + \frac{10D_0}{V} + \frac{2\pi + 3}{3} \frac{d \ln V}{ds}$$

where the first term on RHS describes the effect of magnetic moment conservation in magnetized plasmas, the second one describes the effect of electron - whistler wave interactions, and only the third term describes the pure thermodynamic reaction of the electron fluid as we had derived it in the section ahead. This equation has been derived by Fahr and Dutta-Roy on the basis of kappa-type electron distribution functions selecting exclusively isentropic solutions for the electron plasma flow [8].

This differential equation can then be integrated and leads to the following more completed solution for the electron pressure in case the electron pressure dominates:

$$P_e(s) = P_{e0} \left(\frac{B}{B_0}\right)^{4/3} \left(\frac{V}{V_0}\right)^{\frac{2\pi+3}{3}} \exp\left[10D_0 \int_{s_0}^s \frac{ds}{V}\right] + C_e^s$$

The solution shows that the electron pressure under isentropic conditions decreases with the plasma bulk velocity proportional to  $V(s)^{\frac{2\pi+3}{3}} = V(s)^{3.09}$  whereas under incompressibility conditions it decreases proportional to  $V(s)^2$ . The above relation, however, furthermore shows that, in addition to that dependence on the plasma bulk velocity, dependences on the magnitude of the frozen-in magnetic fields  $B = B(s)$  along the streamlines enforcing the conservation of magnetic particle moments, and wave-electron diffusion, if at all operating, may independently and additionally modify the electron pressure along the streamline. However, for this most general case an adequate and consistent two-fluid MHD solution is not yet available up to the present date. Only for the simplest case of non-magnetic two-fluid solution without wave-particle heating of the electrons one can easily derive from the third remaining term in the upper equation that the dominant electron pressure when experiencing with increasing streamline coordinate  $s$  a decrease of the downstream bulk velocity  $V(s)$  will lead to a decreasing electron pressure, till the electron pressure finally is not anymore dominant over the proton pressure.



**Figure 1:** Consistent MHD-pattern of plasma flow lines in the heliosphere for the case of a mono-fluid proton plasma, scaled in units of AU.

### Conclusions

In earlier papers we have shown that classical monofluid MHD theory delivers straightforward and consistent MHD solutions for the magnetic field configuration and the plasma flow in the heliosheath, both for the upwind case for streamlines approaching the region near the heliopause stagnation point, and for the downwind case for streamlines leading into the heliospheric tail region [14,15]. Here in this article we do demonstrate now, however, that monofluid solutions in fact cannot be accepted as valid solutions of the given problem in the heliosheath region, because it turns out that electrons beyond the solar wind termination shock develop their own independent pressures which are comparable with or even dominant over the proton pressures. This requires a two-fluid representation of the plasma flow system in the heliosphere. Under these conditions the electron pressures become a dynamically relevant quantity which strongly co-influences the resulting plasma dynamics, i.e. a two-fluid treatment of the plasma flow is definitely required here [16-52].

In order to be able to describe electrons and protons as independent, but coupled fluids, one, however, has to pay a look on the kinetic level of the underlying plasma system and had to derive kinetic transport equations for electrons and protons describing the evolution of their kinetic distribution functions along streamlines. When converting them into pressure transport equations, one can arrive at independent solutions for the pressures of electrons and protons as functions of the streamline coordinate  $s$ . In this paper here we present solutions for the MHD plasma flow under the special condition that the electron pressure dominates over the proton pressure which is shown to be the case immediately downstream of the solar wind termination shock, but at decreasing plasma bulk velocities electron pressures fall down and may finally be comparable or even be lower than proton pressures.

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