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Cosmic Baryon Kinetics Expressed by Its Velocity Moments over Times after the Matter Recombination

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Abstract
As we are going to show here it is not easily understandable how cosmic gases like H-atoms, after the recombination of cosmic matter, do thermodynamically behave under the ongoing Hubble-like expansion of the universe. The question namely is not easy to answer; how cosmic gas atoms do in fact recognize the expansion of cosmic 3- space. Contemporary mainstream cosmology takes for granted that gas atoms do react polytropically or even adiabatically to cosmic volume changes and thus do get more and more tenuous and colder in accordance with gas- and thermo- dynamics. However, one has to face the fact that cosmic gases at the recombination era are already nearly collisionless over scales of 10 AU, and how gases react to cosmic volume changes under such conditions is not a trivial problem. We derive in this article a kinetic transport equation which describes the evolution of the gas distribution function f(t, v) in cosmic time t and velocity space of v. This partial differential equation does not allow for a solution in form a separation of the two variables t and v, but instead we can find solutions for two moments of f(v, t), i.e. the density n(t) and the pressure P(t). Then we show that using kappa-like functions for the cosmic gas we can derive such functions as function of their velocity moments, i.e. as functions of cosmic time. It means we understand the kinetic evolution of the cosmic gas by understanding the evolution in cosmic time of their moments.

A Brief View on the Cosmic Matter Recombination Phase

It is generally assumed that before the phase of matter recombination (say about 380000 years ago) matter and radiation were in perfect thermodynamic equilibrium, implying that protons in this phase are described by Maxwell distributions $f(v, t_0) = Max(v, T_0)$ and photons are distributed according to a Planckian black body spectrum for a common temperature T0. A deeper look into the kinetic theory of the physical processes close to and just after the recombination phase of electrons and protons, makes evident that in a homologously expanding universe the baryon distribution function cannot be expected to maintain its Maxwellian shape, since its most relevant velocity moments, i.e. the density and the temperature, vary in an unexpected nonclassical, non-adiabatic manner [1,2]. As consequence of that the entropy of baryons, i.e. of H-atoms, in fact does change with cosmic time, in contrast to the standard thermodynamical expectation.

We start with a brief look on the phase of cosmic electron - proton recombinations thought to have occured at about 380000 years after the so-called Big-Bang, when the temperatures of the cosmic plasma dropped to below 4000 K [3]. It is assumed that at this phase electrons and protons are dynamically and physically tightly coupled to each other, since undergoing strong and frequent mutual interactions both by Coulomb collisions and by Compton collisions with photons. Under such prerequisites a pure thermodynamical equilibrium state seems to be guaranteed, implying that protons and electrons are distributed in velocity-space according to a Maxwellian velocity distribution, and photons maintain a Planckian blackbody spectrum in frequency. Looking at this relevant point more in detail makes it, however, by far not so evident that these assumptions are really fulfilled during this period, mainly because photons and particles react very differently to the cosmological expansion. Photons generally are cooling due to permanently being cosmologically redshifted [4-6]. In contrast particles are not directly feeling the expansion of the universe, unless they feel it adiabatically by mediation of the changing thermodynamic conditions through numerous Coulomb collisions.

Over distances D where the cosmic gas atoms can be considered as collision-free, i.e. for $D \leq \lambda_c$ (with λ_c denoting the actual mean free path with respect to elastic collisions), they will not feel the expansion at all. Only beyond, at distances $D > \lambda_c$, those atoms with velocities larger than $v \leq \lambda_c$ H (i.e. the critical Hubble drift!) are touching the "collisional wall" of their cosmic environment and will start recognizing the cosmic expansion, while others with $v > \lambda_c$ H are not touching this wall. Hereby the expansion of the universe is described by the Hubble parameter with $H=\dot{R}/R$, where R denotes the scale of the universe, and $\dot{R}$ its derivative with respect to cosmic time t. Or expressing it in other words, if one expands the walls of a collision-free gas with a supersonic velocity $V>>v_s$, then this gas will not recognize the expansion, only the few particles of the gas distribution function with velocities $v> v_s$ can interact with the wall and can react "adiabatically" by returning to the system with reduced energy.

Furthermore an additional problem occurs, since Coulomb collisions redistributing velocities among particles and reconstituting the distribution function have a specific property aggravating things in this context. Namely the fact that Coulomb collision cross sections are strongly dependent on the relative velocity w of the colliding particles since being proportional to ($1/w^4$) [8]. This has the consequence that high-velocity particles are much less collision-dominated compared to low-velocity ones. The latter even behave as Collison-free at supercritical large velocities $v > v_c$. So while the low-velocity branch of the distribution thus may still cool adiabatically like a collision-dominated gas and thus feels and reacts to the cosmic expansion in an adiabatic form, the high-velocity branch in contrast behaves collision-free and hence changes in a different, yet unspecified form.

This violates the concept of a joint equilibrium temperature and of a resulting mono-Maxwellian velocity distribution function, and means that there may be a critical evolutionary phase of the universe, due to different forms of cooling in the low- and high-velocity branches of the particle velocity distribution function. Such a situation does not permit the endurance of a Maxwellian distribution to later cosmic times. Hence we shall now look into this interesting evolutionary expansion phase a bit deeper and try to draw some first conclusions concerning the cosmic gas behaviour in the post-recombination era. We shall also demonstrate here that the realistic behaviour of cosmic gases during this phase and later depends on the specific form of the Hubble expansion of the universe, especially an accelerated expansion phase as is often discussed nowadays will strongly influence the thermodynamics of the cosmic gas, creating so-called "over-Maxwellian"- depletions of high velocity particles, i.e. distributions with strongly extinguished high-velocity particles. Such types of functions we shall describe in the forthcoming sections of this paper.

Derivation of the Kinetic Transport Equation for Cosmic Gases

We start out from the generally accepted assumption in modern cosmology, that during the collision-dominated phase of the cosmic evolution, just before the time of matter recombination, matter and radiation, due to frequent energy exchange processes, are in complete thermodynamic equilibrium, i.e. matter and radiation temperatures are identical $T_m = T_s = T_0$. In the following cosmic evolution this equilibrium, however, will experience perturbations as had already been emphasized in the section above and earlier by Fahr and Loch (1991). The upcoming part of the paper shall demonstrate now that, even if a Maxwellian distribution would actually prevail at the entrance to the collision-free cosmic expansion phase, it would not persist at times there after. Just after the recombination phase when electrons and protons recombine to H-atoms, and photons start propagating through cosmic space practically without further interaction with matter, the thermodynamic contact between matter and radiation further on is abolished or switched off. This is one reason why the initial Maxwellian atom distribution function would not persist in the universe during the ongoing collision-free expansion.

To enlucidate this point let us first consider a collision-free particle population in an expanding, spatially symmetric Robertson-Walker universe. Hereby it is clear that due to the cosmological principle and, connected with it, the requirement of spatial homogeneity, also the velocity distribution function of the particles must be isotropic in v and independent on the local cosmic place x. Thus, it must be of the following general form

$$f(v,t) = n(t) \cdot \bar{f}(v,t) \qquad \#$$

Where n(I) denotes the time-variable, cosmic density, only depending on the world time t, and $\bar{f}(v, t)$ is the normalized, time-dependent, isotropic velocity distribution function with the property: $\int \bar{f}(v, t)d^3v=1$. If we now do take into account that particles, moving freely with their velocity v into their $\vec{v}$-associated direction over a distance l, at their new place have to restitution the actual cosmic distribution there, despite the differential Hubble flow and the explicit time-dependence of f, then a locally prevailing co-variant distribution function $f(v', t')$ must exist such with the property that the two associated functions $f(v', t')$ and $f(v, t)$ are related to each other in a Liouville-conform way [9]. To quantify this required relation needs some special care, since particles that are freely moving in a homologously expanding Hubble universe, do in this specific case at their motions not conserve their associated phasespace volumes $d^6\phi = d^3vd^3x$ as they usually do in gas dynamics, since in a homologously expanding cosmic space no particle Lagrangian $L(v, x)$ does exist, as usually does in gas dynamics, and thus no Hamiltonian canonical relations of their dynamical coordinates v and x are valid.

As consequence Liouville's theorem does not require that the differential 6D-phase space volumes $d^6\phi$ are identical, but that the conjugated differential phase space densities are identical to guarantee particle conservation [10]. This is expressed by the following relation:

$$f'(v',t')d^3v'd^3x' = f(v,t)d^3vd^3x \qquad \#$$

When arriving at the place x´ these particles, after passage over a distance l are incorporated into a particle population which has a relative Hubble drift with respect to the origin of the particle given by $v_H = l \cdot H$, co-aligned with $\vec{v}$. Thus the original particle velocity v registered at the new place x´ appears locally tuned down to $v' = v - l \cdot H$, since at the present place x´, deplaced from the original place x by the increment l, all velocities have to be judged with respect to the new local reference frame (standard of rest) with its differential Hubble drift of $(l \cdot H)$ with respect to the particle´s origin.

If all of that is taken into account, it can be shown that one finally is lead to the following kinetic transport equation [1, 2]:

$$\frac{\partial f}{\partial t} = vH \cdot \left(\frac{\partial f}{\partial v}\right) - H \cdot f$$

Which should enable one to derive the resulting distribution function as function of the velocity v and of the cosmic time t. As it was shown already by Fahr (2021), the above kinetic transport equation does not allow for a solution in the form of a separation of variables, i.e. putting $f(v, t)= f_t(t) \cdot f_v(v)$, but one rather needs a different, non-straightforward method of finding a kinetic solution of this above transport equation Equ.(1) [1, 2].

Cosmic Kappa-Functions

One way which may prove to be promising here, is to think of kappa-functions as the underlying distribution functions at cosmic times after the matter recombination. These latter functions a priori have the advantage of covering all kinetic function phenomena spanned between pure power law functions and pure Maxwellian functions which have to be expected at times after matter recombination in the universe *($t \geq t_0$!)* [11, 2 and 10]. Let us therefore now have a look on this latter type of functions with respect to its applicability in cosmology.

Starting from an isotropic kappa-distribution in the frame of the plasma bulk motion which latter has to dissappear anyway in a Robertson-Walker universe (i.e. due to the cosmological principle requiring full 3D- space symmetry!). Local bulk motions would evidently violate this cosmological principle. These types of required functions are generally given in the following form [10].

$$f_\kappa(v) = \frac{n}{\pi^{3/2}\kappa^{3/2}\Theta^3}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}[1+\frac{v^2}{\kappa\Theta^2}]^{-(\kappa+1)}$$

Here n denotes the particle density, K and θ denote two independent, typical kappa-function parameters, and $\Gamma=\Gamma(x)$ means the well-known mathematical Gamma-function. The above distribution function $f_k(v)$ is typical for deviations from the normally expected thermodynamical, collision-dominated equilibrium situation which latter would be characterized by a Maxwellian distribution and would automatically be contained in the upper function family with the case $k \rightarrow \infty$.

Calculating now on the basis of the above distribution function $f_k(v)$ the associated pressure moment Pk, by carrying out the necessary velocity-space integration, then leads to the following [1, 2]:

$$P_\kappa = \frac{4\pi m}{3}\int_0^\infty f_\kappa(v)v^4dv = \frac{m}{2}n\Theta^2\frac{\kappa}{\kappa-3/2}$$

with m denoting the particle mass. This then shows, however, that kappa distributions with kappa-function parameters k and Θ nevertheless do lead to the same pressure moment P_k (i.e. isobaric functions!), – if! the k associated parameter Θ (i.e. the "thermal" spread of the function) is a specific function of k, i.e. $\Theta=\Theta(k)$, and if! this function $\Theta(k)$ is given through the following relation:

$$\Theta^2(\kappa) = 2P_\kappa\frac{\kappa-3/2}{mn\kappa} = \Theta^2_{\kappa,M}\frac{\kappa-3/2}{\kappa}$$

This then opens up another possibility, or if preferred an other way around, one namely can keep P_k as a function parameter of the distribution function and can express Θ as function of the remaining function parameters k, n_k, P_k in the form:

$$\Theta^2(\kappa,n,P) = 2P_\kappa\frac{\kappa-3/2}{mn\kappa} = \frac{2P_\kappa}{mn_\kappa}\frac{\kappa-3/2}{\kappa}$$

This for instance is generally practised in writing Maxwellians $Max(v)$ as functions of their two velocity moments n_{Max} and $T_{Max} = P_{Max}/(Kn_{Max})$ in the form:

$$Max(v) = n_{Max}\frac{1}{(\pi T_{Max})^{3/2}}\exp[-\frac{mv^2}{KT_{Max}}]$$

In this sense, the above kappa-type distribution function could as well be expressed through its parameter k and the function moment's nk and P_k in the form:

$$f_\kappa(v) = \frac{n_\kappa}{\pi^{3/2}\kappa^{3/2}(\frac{2P_\kappa}{mn_\kappa}\frac{\kappa-3/2}{\kappa})^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}[1+\frac{mn_\kappa v^2}{2P_\kappa(\kappa-3/2)}]^{-(\kappa+1)}$$

Now it turns out from a recent paper that, prior to the knowledge of the distribution function $f_k(v)$ itself, one can show that the moments of the above function, starting from the kinetic transport equation Equ.(1) for gases in an expanding universe, can be found without having available the solution of this kinetic transport equation first [1, 2]. From the corresponding moment transport equations of this equation the moments $n_k(t)$ and $P_k(t)$ can be derived, and with the Hubble constant $H_0 = \dot{R}_0/R_0$ (the problem of treating the Hubble parameter as a constant will be discussed in the next section) , lead to the following results for the time-dependence of these moments [1, 2]:

$$n_\kappa = n_{\kappa 0}\exp[-4H_0(t-t_0)]$$

and:

$$P_\kappa(t) = P_{\kappa 0}exp[-6H_0(t-t_0)]$$

This requires prior to solving Equation (1) that the kinetic distribution function, whatever form it has, has to obey the following fact:

$$\frac{P_\kappa(t)}{n_\kappa(t)} = \frac{P_{\kappa 0}}{n_{\kappa 0}}\exp[-2H_0(t-t_0)]$$

If we now take this knowledge and introduce it into the upper kappa-function we then obtain the following form for it:

$$f_\kappa(v,t) = \frac{n_{\kappa 0}\exp[-4H_0(t-t_0)]}{\pi^{3/2}\kappa^{3/2}[\frac{2P_{\kappa 0}}{mn_{\kappa 0}}\exp[-2H_0(t-t_0)]\frac{\kappa-3/2}{\kappa}]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \cdot [1+\frac{mn_{\kappa o}v^2}{2P_{\kappa 0}\exp[-2H_0(t-t_0)](\kappa-3/2)}]^{-(\kappa+1)}$$

or after some mathematical rearrangements:

$$f_\kappa(v,t) = \frac{n_{\kappa 0}\exp[-H_o(t-t_0)]}{\pi^{3/2}\kappa^{3/2}[\frac{2P_{\kappa 0}}{mn_{\kappa 0}}\frac{\kappa-3/2}{\kappa}]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \cdot [1+\frac{\frac{1}{2}mv^2}{(P_{\kappa 0}/n_{\kappa 0})\exp[-2H_0(t-t_0)](\kappa-3/2)}]^{-(\kappa+1)}$$

We now introduce the following quantity; - one could call it: the mean thermal particle energy E0 at the cosmic time $t = t_0$:

$$\frac{P_{\kappa 0}}{n_{\kappa 0}} = \frac{n_{\kappa 0}kT_{\kappa 0}}{n_{\kappa 0}} = E_0$$

and obtain the upper distribution function in the following form:

$$f_\kappa(v,t) = \frac{n_{\kappa 0}\exp[-H_0(t-t_0)]}{\pi^{3/2}\kappa^{3/2}[\frac{2E_0}{m}\frac{\kappa-3/2}{\kappa}]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}[1+\frac{\frac{\frac{1}{2}mv^2}{E_0}\exp[2H_0(t-t_0)]}{(\kappa-3/2)}]^{-(\kappa+1)}$$

Or when expressing, since being more practical, the mean thermal energy by $E0 = (1/2)mv0^2$ one obtains:

$$f_\kappa(v,t) = \frac{n_{\kappa 0}\exp[-H_0(t-t_0)]}{\pi^{3/2}\kappa^{3/2}v_0^3[\frac{\kappa-3/2}{\kappa}]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}[1+\frac{\frac{v^2}{v_0^2}\exp[2H_0(t-t_0)]}{(\kappa-3/2)}]^{-(\kappa+1)}$$

And thus finally obtaining the differential velocity space density, with introduction of the normalized variable $x = v/v0$, by

$$f_\kappa(x,t)x^2dx = \frac{n_{\kappa 0}\exp[-H_0(t-t_0)]}{\pi^{3/2}[\kappa-3/2]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \cdot [1+\frac{x^2\exp[2H_0(t-t_0)]}{(\kappa-3/2)}]^{-(\kappa+1)}x^2dx$$

The above function is essentiallly well defined concerning its v - and t - dependencies, - up to the missing knowledge on the time-dependence of the parameter $x = x(t)$. Assuming, however, the prevalence of a Maxwellian distribution at time $t = t_o$ to would imply that $x(t_0) = x_0 \geq 10$, and then expecting for later cosmic times $t = t_o$ due to the Hubble-drift influence more low-velocity-loaded "over-Maxwellian ´ized" distributions should suggest that the k-parameter perhaps continues to increase according to

$$\mathbf{k(t) = k_0 \exp [H\wedge (t - t_0)]}$$

This then leads to the results shown in Figures 1 and 2.

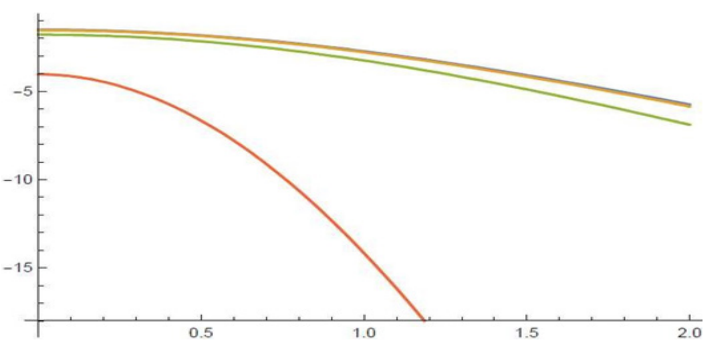

Figure 1: The cosmic baryon distribution function in the times $t_1 = 1year$, $t_2 = 10years$, and $t_3 = 100years$ after the matter recombination at $t = t_0$.

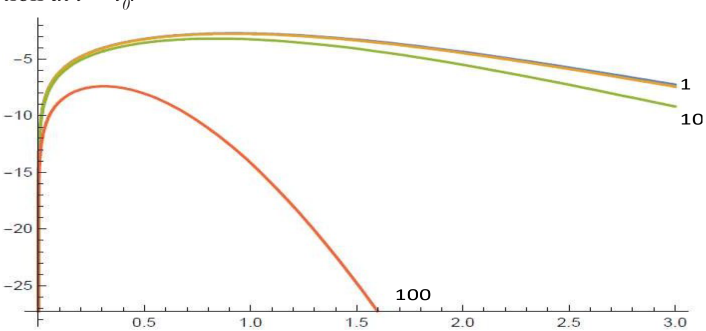

Figure 2: Differential velocity space density of the cosmic baryons at times $t_1 = 1year$, $t_2 = 10years$, and $t_3 = 100years$ after the cosmic matter recombination at time t_0.

In our Figures 1 and 2 we have assumed that the parameter x attains a dependence on cosmic time according to

$$\mathbf{x=x_0 \exp [H_0 (t-t_0)]}$$

with $x_0=10$, and it is shown, how within 1, 10, 100 years the cosmic distribution function would then change its velocity profile starting from a Maxwellian tending to more centrally piled "over-Maxwellians", i.e. just the opposite to non-equilibrium, power law distributions.

The basis hereby in Figure 1 is a Hubble constant of $H_0 = 70km/s/Mpc$ which is confirmed for the present time. If this Hubble constant is used by us for the time after matter recombination $t \geq t_0$, it means and requires that the Hubble constant $H = H_0$ more or less should not have changed since these times till now - at first glance a rather astonishing and audacious assumption. - But astonishingly enough this is in fact a viable assumption as we are going to show now in the next section.

The Hubble Constant in the Early Universe

For Friedman-Lemaitre-Robertson-Walker cosmologies (FLRW) the Hubble parameter H =R ˙/R can be given in form of the following differential equation 1 [5, 11]:

$$H^2 = \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}[\rho_B + \rho_D + \rho_\nu + \rho_\Lambda]$$

where G is Newton's gravitational constant, and ρ_B,ρ_D, ρ_ν, $\rho_\wedge$ denote the equivalent cosmic mass densities of baryons, of dark matter, of photons, and of the vacuum energy. In case all of these quantities do count, then it is complicated to find a solution for H and $R(t)$ over all cosmic times, because ρ_B may vary proportional to R^{-3}, ρ_D most probably also according to R^{-3}, but ρ_ν is generally thought to vary according to R^{-4} [5-7]. Amongst these quantities, the cosmic vacuum energy density $\rho\wedge$ is perhaps physically the least certain quantity, but if it is described with Einstein´s cosmological constant $\wedge$, then it represents a positive, constant energy density, i.e its mass equivalent $\rho\wedge$ hence would as well be a positive constant quantity.

From recent supernova SN1a observations it has been concluded that at the present cosmic era and most probably already sometimes ago we were and are in an accelerated expansion phase of the universe, expressing the fact that $\rho\Lambda$ is the dominant quantity amongst the upper ingredients in the universe. If this can be taken as the truth also back to the times of matter recombination, then in fact we can assume that the above differential equation can be written in the much more simplified form:

$$H_\Lambda = \frac{\dot{R}}{R} = \sqrt{\frac{8\pi G}{3}[\rho_B + \rho_D + \rho_\nu + \rho_\Lambda]} \simeq \sqrt{\frac{8\pi G}{3}[\rho_\Lambda]} = const$$

in fact then describing the expansion of the universe by the expression:

$$R(t) = R_0 \exp[\sqrt{\frac{8\pi G}{3}[\rho_\Lambda]}\,(t-t_0)] = R_0\exp[H_\Lambda(t-t_0)]$$

Taking the above result and reminding the result that we derived in the section before for the first moment of the baryon distribution function, i.e. the density nk(t), given by:

$$n_\kappa(t) = n_{\kappa 0}\exp[-4H_\Lambda(t-t_0)] \ = n_{\kappa 0} \cdot (R(t)/R_0)^{-4}$$

We obtain a somewhat astonishing result, meaning that in an acceleratedly expanding universe like the one with $H = H_\Lambda$ the local density is falling off with the inverse of the fourth power of the scale of the universe. This should mean that the total mass MU of the universe is not constant, but decreasing like:

$$\frac{dM_U}{dt} = \frac{d}{dt}[\frac{4\pi}{3}R^3\rho] = \frac{4\pi}{3}[3\rho_0\frac{R_o^4}{R^2} - 4\rho_0\frac{R_0^4}{R^2}] = -\frac{4\pi}{3}\rho_0\frac{R_0^4}{R^2}$$

However, the reader must be warned, since the concept of a total mass M_U of the universe is by far not clearcut, it rather must be deeply discussed how precisely the meaning of M_U should defined. It turns out that it must be understood as the value of all masses "instantaneously or simultaneously" surrounding each arbitrary point in the FLRW- universe and its precise formulation leads to unexpected complications [13-16]. So for instance in Fahr and Heyl it leads to the following expression

$$M_U(t) = 4\pi\rho(t)\int_0^{R_u}\frac{r^2}{\sqrt{1-\frac{8\pi G}{rc^2}\rho(t)\int_0^r x^2dx}}dr$$

and evaluates to:

$$M_U(t) \simeq \frac{c^2}{G}R_U(t)$$

expressing the fact that the "so-called" total mass of the universe has a Machian character and increases with the size R_U of the universe. If therefore it could be conclud $\rho(t) = \rho_o \cdot (R_U(t)/R_{U0})^{-3}$. cle increases in the same way as the mass of the universe, then the mass density is again falling off with
and no problem remains.

Conclusions

In the aforegoing sections of this paper we have started from the kinetic transport equation (Equ. (1)) for the distribution function *f(v, t)* of a baryon gas embedded in the cosmic FLRWspace- time metrics of an expanding universe. We first could show that this differential equation does not allow for a solution by separation of the variables in the form $f(v, t) = f_v(v) \cdot f_t(t)$, but could demonstrate that the kinetic transport equation Equ.(1) allows to derive solutions for two of its velocity moments , namely the baryon density $n_k(t)$ and the baryon pressure $p_k(t)$, prior to the solution of *f(v, t)* itself. Based on the knowledge we have then presented the kinetic distribution function in form of a general isotropic kappa-function $f(v, t) = f_k(v, x(t), \Theta(t))$ that by use of its already known moments then can be written in the form $f(v, t) = n_k(t). f_k(v, k(t), P_k(t))$. As we can show here, to overcome the Hubble drift between two reference points bridged by moving baryons in the expanding universe, high velocity branches of the distribution function are systematically suppressed, and the velocity spread of the distribution function decreases with increasing cosmic times t, a phenomenon which we may call "super-Maxwellisation". This is seen in Figures 1 and 2 showing the resulting distribution function for times $t_1 = 1year$, $t_2 = 10years$, and $t_3 = 100years$ ars after the time t_0 of the cosmic matter recombination. The cosmic particles with increasing cosmic times are systematically more concentrated at the low velocity region of velocity space, which is also described by the temperature decrease with time according to the result derived from the moments:

$$kT_\kappa(t) = P_\kappa(t)/n_\kappa(t) = kT_0\exp[-H_\Lambda(t-t_0)]$$

Telling that in an expansing universe with a constant Hubble-constant H_Λ the co smic gas temperatures $T_k(t)$ should permanently decrease and finally even fall down to the absolute zero-point [17-20].

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