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Research Article

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# Terminal Speed of an Electron Accelerated by an Electric Field

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## Abstract
*An electron of charge -e and mass m moving at time t with speed v and acceleration (dv/dt) in a straight line, in an electric field of magnitude E, comes under the influence of an impressed force -eE. The moving electron encounters a radiation reaction force eEv/c so that the accelerating force becomes -eE(1 – v/c) = -m(dv/dt), with constant mass m. It emits radiation with radiation power as (eEv^2)/c and reaches a terminal speed equal to that of light c, whereby the impressed force -eE becomes equal and opposite to the radiation reaction force and the acceleration reduces to zero. It is emission of radiation, not increase of mass with speed, that prevents an accelerated electron from going beyond the speed of light. Doing away with infinitely large masses of electrons moving at the speed of light in linear particle accelerators, should bring great relief to physicists all over the world.*

**Keywords:** Acceleration, Force, Electric Charge, Electric Field, Electron, Mass, Speed, Special Relativity.

## Introduction
In a most remarkable experiment in 1964, William Bertozzi [1] of the Massachusetts Institute of Technology, demonstrated the existence of a universal limiting speed, equal to the speed of light in a vacuum. The experiment showed that electrons accelerated through energies of 15 MeV or over, attain, for all practical purposes, the speed of light c.

The theory of special relativity [2, 3, 4] explains the existence of a limiting speed equal to the speed of light c by positing that mass of a moving charged particle, such as an electron, increases with its speed, becoming infinitely large at the speed of light. Since an infinitely large mass cannot be accelerated any faster by a finite force, that speed is a limit.

This paper shows that the accelerating force ***–e*E** exerted by an electric field of intensity E on an electron of charge -e, accelerated in a straight line, decreases with speed v. The electron encounters a radiation reaction force = *e***E***v/c* = -*e***E***v/c*, and it emits radiation with power (*eE**v^2***)/c. The acceleration reduces to zero at the speed of light c whereby the radiation reaction force becomes equal and opposite to the acceleraying force. Infinite mass or zero force at the speed of light, leads to zero acceleration and constant speed c as a limit, in accordance with Newton's second law of motion. It is unfortunate that the theory of special relativity chose *mass* as the variable rather than *force*. The ultimate speed, without infinite mass, is a more comfortable and realistic proposition [5].

The paper deals with rectilinear motion of an electron, accelerated by an electric field, under classical electrodynamics, relativistic electrodynamics, and radiative electrodynamics. In radiative electrodynamics, a charged particle moves, in an electric field, at constant mass, as the rest mass, and with emission of radiation.

### Classical electrodynamics
In classical electrodynamics, the accelerating force **F** on an electron of charge *-e* and mass m moving at time *t,* with velocity **v**, in a straight line, is given by vector equation:

$$\mathbf{F} = -e\mathbf{E} = m\frac{d\mathbf{v}}{dt} \qquad 1$$

For an accelerated electron, equation (1) gives the scalar equation:

$$eE = m\frac{dv}{dt} \qquad 2$$

The solution of equation (2) for an electron accelerated by a constant electric field E, from zero initial speed, is:

$$v = at \qquad 3$$

where a = *eE/m* is a constant.
For a decelerated electron, equation (1) gives the scalar equation:

$$eE = -m\frac{dv}{dt} \qquad 4$$

The solution of equation (4) for an electron decelerated by a constant electric field *E*, from the speed of light c, is:

$$v = c - at \qquad 5$$

A graph of *v/c* against at/c is shown as straight lines *A1* and *A2* in Figure 1, for equations (3) and (5) respectively.

### Relativistic electrodynamics

Under relativistic electrodynamics, accelerating force **F** on an electron of charge -e and velocity dependent mass *m* moving at time *t*, with velocity **v**, in a straight line, is:

$$\mathbf{F} = -e\mathbf{E} = \frac{d(m\mathbf{v})}{dt}$$

The scalar equation for an electron accelerated in a straight line, is:

$$eE = \frac{d(mv)}{dt} \qquad 6$$

where *m* is given by the relativistic mass-velocity formula, with $m_o$ as the rest mass:

$$m = m_o\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \qquad 7$$

Equation (6) and (7), for an electron accelerated from 0 initial speed, by a constant electric field *E*, give:

$$\frac{eE}{m_o}t = at = v\left\{\left(1 - \frac{v^2}{c^2}\right)\right\}^{-\frac{1}{2}}$$

$$v = at\left\{1 + \left(\frac{at}{c}\right)^2\right\}^{-\frac{1}{2}} \qquad 8$$

A graph of *v/c* against at/c is shown as dashed curve *B1* for equation (8) and dashed horizontal line *B2* in Figure 1. In relativistic electrodynamics, an electron moving at the speed of light, cannot be stopped by any force. It continues to move at the same speed, losing potential energy without gaining kinetic energy.

### Acceleration in Radiative Electrodynamics

An electron of mass *m* and charge *–e* moving at time *t* with velocity **v** and acceleration (*dv/dt*) in the opposite direction of an electric field of intensity E, encounters a radiation reaction force *–eE***v***/c* = *e***E***v/c.* The accelerating force **F** on the electron, equal to the sum of the impressed force *–e***E** and the radiation reaction force *e***E***v/c,* that is *–e***E**(1 – v/c), is given by Newton's second law of motion, as vector equation:

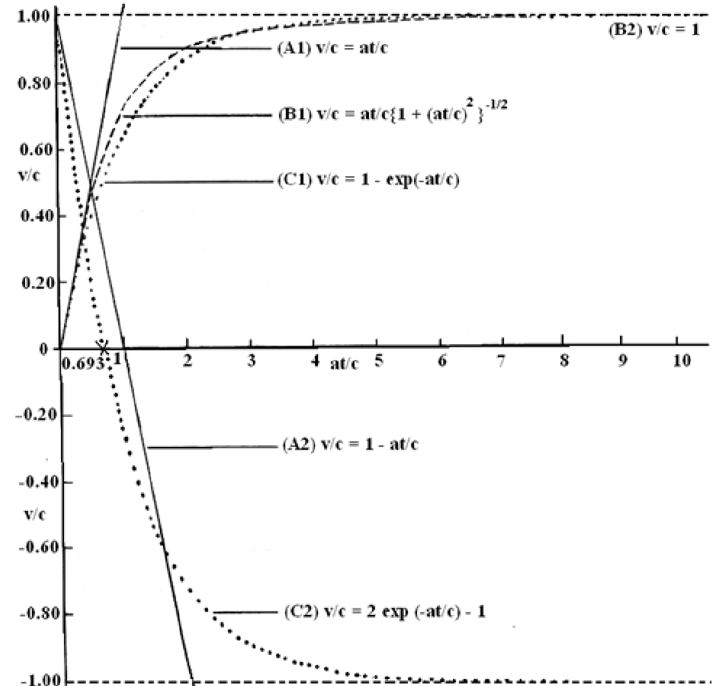


**Figure 1:** Graph of *v/c* (speed in units of c) against *at/c (time in units of c/a)* for an electron of charge *–e* and mass *m = m* accelerated from zero initial speed or decelerated from the speed of light c, by a uniform electrostatic field of magnitude E, where *a = eE/m*; the lines (A1) and (A2) according to classical electrodynamics, the dashed curve (B1) and line (B2) according to relativistic electrodynamics and the dotted curves (C1) and (C2) according to equations 12 and 15 of radiative electrodynamics.

$$\mathbf{F} = -e\mathbf{E}\left(1 - \frac{v}{c}\right) = m\frac{d\mathbf{v}}{dt} \qquad 9$$

Since velocity **v**, of magnitude v, is in the opposite direction of **E,** the scalar equation is:

$$F = -eE\left(1 - \frac{v}{c}\right) = -m\frac{dv}{dt} \qquad 10$$

where *c* is the speed of light in a vacuum and *m* is the mass of the particle, which is considered as independent of speed *v* of the particle.

For acceleration in a uniform field (*E* constant), where *eE/m* = a is a constant, the solution of equation (10), for an electron accelerated from an initial speed u, is:

$$v = c - (c - u)\exp\left(\frac{-at}{c}\right) \quad 11$$

For an electron accelerated from an initial speed $u$ = 0, equation (11) becomes:

$$\frac{v}{c} = 1 - \exp\left(-\frac{at}{c}\right) \quad 12$$

In equation (12), the speed of light c is the limit to which a charged particle, such as an electron, can be accelerated by an electric field. A graph of v/c against at/c is shown as curve C1 in Figure 1 for equation (12).

### Deceleration in radiative Electrodynamics

For a decelerated electron the differential equation of motion (replacing v by –v in equation 10) becomes:

$$F = -eE\left(1 + \frac{v}{c}\right) = m\frac{dv}{dt} \quad 13$$

The solution of equation (13), for a charged particle decelerated, by a uniform electric field, from speed $u$, is:

$$v = (c + u)\exp\left(-\frac{at}{c}\right) - c \quad 14$$

For an electron decelerated from the speed of light c, equation (14) becomes:

$$\frac{v}{c} = 2\exp\left(-\frac{at}{c}\right) - 1 \quad 15$$

In equations (15), the particle is decelerated to a stop and then accelerated in the opposite direction to reach a terminal speed equal to *–c*, shown as *C2* of the graphs in Figure 1. This result is not obtainable from the point of view of the theory of special relativity. In special relativity a particle moving at the speed of light will continue to move at that speed, losing potential energy without gaining kinetic energy.

### Speed-time Equations and radiation of energy.

Figure 1 is a graph of v/c (*speed in units of c*) against *at/c* (*time in units of c/a* ) for an electron of charge *–e* and mass m accelerated from zero initial speed or decelerated from the speed of light c, by a uniform field of magnitude *E*, where $a = eE/m$ ; the lines (A1) and (A2) according to classical electrodynamics, the dashed curve (B1) and line (B2) according to relativistic electrodynamics and the dotted curves (C1) and (C2) according to the radiative electrodynamics giving equations (12) and (15).

In classical electrodynamics, an electron of charge *–e* and mass m, moving at the speed of light c, on entering a uniform retarding field of magnitude *E*, should be stopped in time $t = mc/eE$, energy radiation notwithstanding. In relativistic electrodynamics an electron moving at the speed of light c should be unstoppable by any force. In the electrodynamics advanced in this paper, an electron moving at the speed of light *c* is easily decelerated to a stop, by a uniform retarding field *E*, in time $t = 0.693mc/eE$, with radiation of energy. The energy radiated is equal to the difference between change in potential energy and change in kinetic energy.

## Results and discussion

- Equations (10) and (13) are extensions of Coulomb's law of electrostatic force, taking into consideration the speed of a charged particle in an electric field.
- Equations (12) and (15) give the speed of light c as the ultimate speed with mass of a moving particle remaining constant as the rest mass.
- Figure 1 shows that for low speeds (<0.1c), relativistic electrodynamics and radiative electrodynamics reduce to classical electrodynamics.
- Figure 1 shows that for accelerated electrons, relativistic electrodynamics (curve B1) and radiative electrodynamics (Curve C1) are almost in agreement but differ markedly for decelerated electrons (line B2 and curve C2).
- In radiative electrodynamics (curve C2), an electron can be decelerated from the speed of light c and brought to rest in time 0.693c/a, and may be accelerated in the opposite direction, to an ultimate speed equal to -c.
- No curve is obtainable for electrons decelerated from the speed of light c.
- It is energy radiation which makes all the difference between classical, relativistic and the radiative electrodynamics advanced in this paper.
- The paper introduces radiative electrodynamics, in contrast to classical and relativistic electrodynamics.
- Energy radiated by an electron, moving in an electric field, is the difference between change in potential energy and change in kinetic energy.

## Conclusion

The speed of light c, being an ultimate limit to which a charged particle, such as an electron, can be accelerated by an electric field, has nothing to do with the mass of the particle. It is a property of the electric field and radiation reaction force that limit the speed of an accelerated charged particle, such as an electron, to that of light c as the terminal speed.

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