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The Thermodynamics of Cosmic Gases in Expanding Universes Based on Vlasow-Theoretical Grounds

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Abstract

Contemporary cosmology is taking for granted that before the phase of matter recombination matter and radiation were in perfect thermodynamic equilibrium. This would mean that protons in this phase of the universe are described by Maxwell distributions and photons are described by a Planckian black body law. When looking, however, a bit deeper into the kinetic theory of the physical processes close to and just after the recombination phase of electrons and protons, it becomes evident that in a homologously expanding universe proton distribution functions will not maintain their Maxwellian profile and connected with it, that their most relevant velocity moments, i.e. their density and their temperature, vary in an unexpected nonclassical, non-adiabatic manner. As consequence of that, in contrast to the classical view, the entropy of free atoms does change with cosmic time contrary to the standard thermodynamically expectation. We shall also demonstrate here that the realistic behaviour of cosmic gases in this phase and later depends on the specific form of the Hubble expansion of the universe, especially an accelerated expansion phase as is discussed nowadays will strongly influence the thermodynamics of the cosmic gas.

Introductory remarks on the cosmic matter recombination phase

We may start our considerations here with a brief view on the phase of cosmic electron - proton recombination's thought to have occured at about 380000 years after the so-called Big-Bang, when the temperatures of the cosmic plasma dropped to below 4000 K (see e.g. Partridge, 1965, or Fahr and Loch, 1991, Fahr Zoennchen, 2009). In standard main-stream cosmology it is tacitly assumed that at this cosmic recombination phase electrons and protons are dynamically and physically tightly bound to each other and undergo strong mutual interactions both by Coulomb collisions and by Compton collisions. With these conditions taken for granted, a pure thermodynamically equilibrium state would appear to be guaranteed. This implies that protons and electrons are kinetically distributed in velocity-space according to Maxwellian velocity distributions, and photons have a Planckian blackbody distribution in frequency. Looking a little bit more in detail on this relevant point, it is, however, by far not so evident that these assumptions really can be expected to be fulfilled during this period of cosmic evolution. This is because photons and particles are reacting very differently to the cosmological expansion; photons generally are considered to be cooling due to permanently being cosmologically redshifted (see e.g. Peacock, 1999, Goenner, 1996). Contrary to that, particles in first order are not directly feeling the expansion of the universe, unless they feel it adiabatically by mediation through numerous Coulomb collisions. Over distances D where the cosmic gas can be considered as collision-free, i.e. for $D \leq \lambda_c$ (with λ_c denoting the actual mean free path with respect to elastic collisions), they will not feel the expansion at all, only beyond, at distances $D > \lambda_c$, those atoms with velocities larger than $v \geq \lambda_c \bullet H$ are touching the "collisional wall" of their cosmic environment and will start recognizing the cosmic expansion. Hereby the expansion of the universe is described by the Hubble parameter with $H = \dot{R}/R$, where R denotes the scale of the universe, and $\dot{R}$ its derivative with respect to cosmic time t. In other words, if one expands the walls of a collision-free gas with a supersonic velocity $V >> v_s$, then this gas will not recognize the expansion, only those few particles of the gas distribution function with velocities $v > v_c$ will interact with the wall and thus can react "adiabatically" by returning to the system with reduced energy.

In the case of a gas included in a box with subsonic expansion of its walls most of the particles would recognize this expansion. To say it in brief: The expanding walls with an expansion velocity V should keep in touch with the particles of the system, meaning that slow particles with particle velocities $v < V$ do not feel the expansion since not interacting with the moving walls, while those with velocities $v > V$ feel it, because their reflection velocities v′ when coming back from the wall is reduced, i.e. $v' < v$. There is an additional problem occurring, since Coulomb collisions redistributing velocities among particles have a specific property which makes things highly problematic in this context. This is the fact that Coulomb collision cross sections are strongly dependent on the relative velocity w of the colliding particles, namely being proportional to ($1/w^4$) (see Spitzer, 1956). This evidently has the consequence that high-velocity particles are much less collision-dominated compared to low-velocity ones, they may even be considered as collison-free at super critically large velocities $v > v_c$. So while the low-velocity branch of the distribution thus may still cool adiabatically like a collision-dominated gas and thus feels and reacts to the cosmic expansion in an adiabatic form, the high-velocity branch in contrast behaves collision-free and hence changes in a different form. This violates the concept of a joint equilibrium temperature and of a resulting mono-Maxwellian velocity distribution function, and means that there may be a critical evolutionary phase of the universe, due to different forms of cooling in the low- and high-velocity branches of the particle velocity distribution function. This does not permit the persistence of a Maxwellian equilibrium distribution to later cosmic times. We shall now look into this interesting evolutionary expansion phase a bit deeper and try to draw some first conclusions concerning the cosmic gas behaviour in the post-recombination era.

A Liouville-theoretical approach to the thermodynamics of cosmic gases

We start out from the generally accepted assumption in modern cosmology, that during the collision-dominated phase of the cosmic evolution, just before the time of matter recombination, matter and radiation, due to frequent energy exchange processes, are in a complete thermodynamic equilibrium state. In the following cosmic evolution this equilibrium, however, will experience perturbations as had already been emphasized in the section above and earlier by Fahr and Loch (1991). The upcoming part of the paper shall demonstrate now that, even if a Maxwellian distribution would actually prevail at the beginning of the collision-free cosmic expansion phase, it would not persist thereafter. Just after the recombination phase when electrons and protons recombine to H-atoms, and photons start propagating through cosmic space practically without further interaction with matter, the thermodynamic contact between matter and radiation further on is abolished or switched off. This is one reason why the initial Maxwellian atom distribution function would not persist in the universe during the ongoing collision-free expansion.

To enlucidate this point let us first consider a collision-free particle population in an expanding, spatially symmetric Robertson-Walker universe. Hereby it is clear that due to the cosmological principle and, connected with it, the spatial homogeneity requirement, also the velocity distribution function of the particles must be isotropic in v and independent on the local cosmic place x. Thus it must be of the following general form

$$f(v, t) = n(t) \bullet \bar{f}(v, t) \qquad \#$$

where n(t) denotes the cosmic, time-variable density, only depending on the world time t, and $\bar{f}(v, t)$ is the normalized, time-dependent, isotropic velocity distribution function with the property: $\int \bar{f}(v, t) d^3v = 1$. If we now do take into account that particles, moving freely with their velocity v into their $\vec{v}$-associated direction over a distance l, at their new place have to restitute the actual cosmic distribution there, despite the differential Hubble flow and the explicit time-dependence of f, then a locally prevailing co-variant, but perhaps form-invariant distribution function $f(v', t')$ must exist such that the two associated functions $f(v', t')$ and $f(v, t)$ are related to each other in a very specific, namely Liouville-conform way (see e.g. Cercigniani, 1988, Landau-Lifshitz, 1990).

To quantify this required relation needs some special care, since particles that are freely moving in a homologously expanding Hubble universe, do in this specific case at their motions not conserve their associated phasespace volumes $d^6\Phi = d^3v d^3x$, since in a homologously expanding cosmic space no particle Lagrangian $L(v, x)$ does exist, as usually does in gas dynamics, and thus no Hamiltonian canonical relations of their dynamical coordinates v and x are valid. Hence Liouville's theorem (see e.g Chapman and Cowling, 1952) then does not require that the differential 6D-phase space volumes $d^6\Phi$ are identical, but that the conjugated differential phase space densities are identical to guarantee that no particle losses occur. This is expressed by the following relation:

$$f'(v', t')d^3v' d^3x' = f(v, t)d^3v d^3x \qquad \#$$

When arriving at the place x´ these particles, after passage over a distance l are incorporated into a particle population which has a relative Hubble drift with respect to the origin of the particle given by $v_H = l \bullet H$ co-aligned with $\vec{v}$, where $H = \dot{R}/R$ means the actual Hubble parameter, R denoting the scale of the universe. Thus the original particle velocity v' registered at the new place x' is locally tuned down to $v' = v - l \bullet H$, since at the present place x', deplaced from the original place x by the increment l, all velocities have to be judged with respect to the new local reference frame (standard of rest) with a differential Hubble drift of $l \bullet H$.

Furthermore all dimensions of the space volume within a time intervall $\Delta t = t' - t$ are cosmologically expanded, so that $dx' = dx(1 + H \bullet \Delta t)$ holds. The velocity increments at the passage of the particle from x $\vec{\text{to}}$ x' are only changed in the two dimensions perpendicular to v while in this latter dimension on the inertial trajectory no change of the velocity increment occurs. Hence a complete re-incorporation into the locally valid distribution function then implies, taking into account here the linearizability of small quantities $\Delta t/t = l/tv$ and $\Delta v/v = -l \bullet H/v$, that one can express the above Liouville requirement in the following form:

$$f'(v', t')d^3v' d^3x' = [f(v, t) + \frac{\partial f}{\partial t}\Delta t + \frac{\partial f}{\partial v}\Delta v] \bullet (1 + \frac{\Delta v}{v})^2 (1 + H \bullet \Delta t)^3 d^3v d^3x = f(v, t)d^3v d^3x$$

This then means for terms of first order

$$\frac{\partial f}{\partial t}\Delta t + \frac{\partial f}{\partial v}\Delta v + 2\frac{\Delta v}{v} \bullet f + 3(H \bullet \Delta t) \bullet f = 0$$

and consequently

$$\frac{\partial f}{\partial t}(l/v) - \frac{\partial f}{\partial v}(l \bullet H) - 2\frac{(l \cdot H)}{v} \bullet f + 3H\frac{1}{v} \bullet f = 0 \qquad \#$$

and finally after some simplifying rearrangements leading to the following requirement

$$(\partial f/\partial t) = vH \bullet (\partial f/\partial v) - Hf \qquad (1)$$

Starting at a time $t = t_0$ from a Maxwell distribution with $f_0(v, t_0) = Max(v, t_0)$, one could try to solve the upper partial differential equation numerically using

$$(\partial f_0/\partial t) = vH \bullet (\partial f_0/\partial v) - Hf_0 = H \bullet [(\frac{m}{2\pi K T_0})^{3/2} \exp[-\frac{mv^2}{KT_0}] \bullet [\frac{2mv^2}{KT_0} - 1] \qquad \#$$

and find the solution for the distribution function for all later times $t \geq t_0$.

Evolution of Velocity Moments

However, instead of doing this, we prefer to make use of a much simpler procedure: Namely looking now first for the most interesting velocity moments, like density $n(t)$ and pressure $P(t)$, of the function $f(v, t)$ fulfilling the above partial differential equation of kinetic transport, i.e. Equ. (1), - then by multiplying this upper equation with a) $4\pi v^2 dv$ and b) $(4\pi/3)\, mv^4 dv$ and integrating over the whole velocity space leads to the following relations:

Density

$$(\partial n/\partial t) = 4\pi H \bullet \int v^3 (\partial f/\partial t) dv - Hn \qquad \#$$

which can by partial integration be developed further to:

$$\partial n/\partial t = 4\pi H \bullet [\int_0^\infty \frac{\partial}{\partial v}(v^3 f) dv - \int_0^\infty 3v^2 f dv] - Hn \qquad \#$$

and yields

$$\partial n/\partial t = 4\pi H \bullet [\left|(v^3 f)\right|_0^\infty - \frac{3}{4\pi}\int_0^\infty 4\pi v^2 f dv] - Hn \qquad \#$$

and further

$$\partial n/\partial t = -3H \bullet n - Hn \qquad \#$$

finally leading to:

$$\frac{1}{n}\frac{d}{dt} n = -4H$$

with the solution:

$$n = n_0 \exp[-4H(t-t_0)] \qquad (2) \qquad \#$$

Pressure

Multiplication of the kinetic transport equation by $(4\pi/3)\, mv^4 dv$ and integration over velocity space leads to:

$$(\partial P/\partial t) = \frac{4\pi}{3} mH \int_0^\infty v^5 (\partial f/\partial v) dv - HP$$

and along the same practise already applied before then leads to:

$$\partial P/\partial t = \frac{4\pi}{3} mH[\int_0^\infty \frac{\partial}{\partial v}[v^5 \frac{\partial f}{\partial v}] dv - 5\int_0^\infty v^4 f dv] - HP$$

yielding:

$$\frac{\partial P}{\partial t} = -5\frac{4\pi}{3} mH \int_0^\infty v^4 f dv - HP = -6HP$$

with the solution:

$$P(t) = P_0 \exp[-6H(t-t_0)] \qquad (3) \qquad \#$$

The Entropy of the Cosmic Gas in the Expanding Universe

Taking the above results from Equations (2) and (3), this then immediately makes evident that one for instance finds that the cosmic particle entropy $S = c_v(P/n^\gamma)$ (see e.g. Lifshitz and Pitaevskii, 1995, Serrin, 1959, Erkaev et al., 2000) with c_v and $\gamma = (f + 2)/f = 5/3$ denoting the heat capacity at constant volume and the polytropic gas index, is given by

$$S(t) = P(t)/n(t)^\gamma = (P_0/n_0^\gamma) \exp[-(6-4\gamma)H(t-t_0)] \qquad \#$$
$$= (P_0/n_0^\gamma) \exp[+(2/3)H(t-t_0)] \qquad \#$$

surprisingly enough now turns out to not be constant, but increasing with time t, which means that in fact no adiabatic behaviour of the expanding particle gas occurs, and that as a consequence the gas entropy S(t) is not constant, but inreases with cosmic time as

$$S = S(t) = S_0 \exp[=(2/3)H(t-t_0)] \qquad \#$$

The Hamiltonian case

At this point of our study it is perhaps historically interesting to see that, when assuming the commonly used Hamilton canonical relations to be valid, i.e. $dL/dp_i = dx_i/dt;\ - dL/dx_i = dp_i/dt,$ the Liouville theorem would then, instead of the kinetic transport requirement formulated in Equ. (1), simply require $f'(v', t') = f(v, t)$ and hence would thus lead to the following more simple form of a Vlasow equation

$$(\frac{\partial f}{\partial t}) - vH(\frac{\partial f}{\partial v}) = 0 \qquad \#$$

In that latter case, the first velocity moment now is found from the following relation

$$\frac{\partial n}{\partial t} = 4\pi H \int \frac{\partial}{\partial v}(v^3 f) dv - 12\pi H \int v^2 f dv = -3nH = -3n\frac{\dot{R}}{R} \qquad \#$$

which for the time period of a constant Hubble parameter $H(t) \simeq H_0$ can easily be identified with the solution $n(t) \sim R(t)^{-3}$, i.e. a baryon density falling off inversely proportional to the cosmic volume $V(t) = (4\pi/3)R^3(t)$.

Also in this Hamiltonian case we would obtain the pressure moment P(t) in the form:

$$dP/dt = -5HP \qquad \#$$

which now in this case based on $\gamma = 5/3$ leads to an entropy of

$$S(t)=P(t)/n(t)^\gamma - (Po/n_0^\gamma)\exp[-(5-3(5/3))H(t-t0)] = \text{const} \qquad \#$$

which interestingly enough means that in this “classic Hamiltonian” case both an adiabatic and isentropic expansion is retained.

The "Hamiltonian case", however, is not given under the conditions of a cosmic Hubble-like expansion and thus here is not a valid assumption! - This is due to the fact that the classical Hamilton canonical relations in the cosmic case are not valid due to the non-conservative cosmic Hubble-forces that are acting connected with a homologous Robertson-Walker expansion and lead to the cosmic relation $dp/dt = -pH$.

Would Maxwell stay Maxwell under non-Hamiltonian conditions?

Though we are not aiming here at the full solution of the kinetic transport equation, Equ. (1), we can nevertheless check, whether a Maxwellian distribution of the cosmic gas atoms would prevail, if it at least were present before the recombination phase. We know that under the cosmic conditions the "cosmically correct" Vlasow equation, Eqn. (1), holds and this equation allows to check whether or not an initial Maxwellian velocity distribution function would persist during the ongoing collision-free expansion at times

$t \geq t_0$. In that case one namely finds for $f(v, t) = Max(v, t) \sim n(t) T(t)^{-3/2} exp[-mv^2/2KT(t)]$, with $n(t)$ and $T(t)$ being time-dependent moments of the Maxwellian distributed cosmic particles, that one obtains the two relevant Vlasow derivatives ∂f/∂t and ∂f/∂v in the following form:

$$\frac{\partial f}{\partial t} = f(v,t) \cdot [\frac{\partial}{\partial t} ln(n) - (3/2)(\dot{T}/T) + \frac{mv^2}{2KT}(\dot{T}/T)] \qquad \#$$

and

$$\frac{\partial f}{\partial v} = -f(v,t) \cdot ((mv)/(KT))$$

These two above expressions then lead to the following Vlasow requirement (see Eqn. (1))

$$\frac{\partial}{\partial t} ln(n) - (3/2)(\dot{T}/T) + \frac{mv^2}{2KT}(\dot{T}/T) = -H \cdot [\frac{mv^2}{KT} + 1] \qquad \#$$

In order to validify the above equation obviously the two terms with v^2 have to cancel each other, since $n(t)$ and $T(t)$ are velocity moments of f, hence independent on v. This is evidently only satisfied, if the change of the temperature with cosmic time is given by

$$T = T_0 \cdot exp(-2H(t-t_0)) \qquad \#$$

This above dependence in fact is obtained when inspecting the earlier found solutions for the moments n(t) and P(t) (see Eqns. (2) and (3), because these solutions exactly give

$$T(t) = (P(t)/(Kn(t)) = P_0/(Kn_o^{\gamma}) \cdot exp(-(4-2)H(t-t_0)) = T_0 \cdot exp(-2H(t-t_0)) \qquad \#$$

With that the above requirement then only reduces to the following expression

$$\frac{d}{dt} ln(n(t)) - (3/2)(\dot{T}/T) = -H \qquad \#$$

which then finally leads to the requirement

$$-2H - (3/2)(-2H) = -H \qquad \#$$

making it mathematically evident that this requirement, i.e. +1 = -1, cannot be fulfilled, and thus meaning that consequently a Maxwellian particle distribution cannot be maintained over the times after the recombination, not even at a collision-free cosmic expansion. With this knowledge one could be seduced and encouraged to find out more about the kinetic situation of the cosmic gas under these conditions just after the recombination era. What kind of distribution function $f(v, t > t_0)$ should be expected for that period? An new independent way of looking at this problem is to use the following kinetic transport equation used by Fahr et al. (2016) which for the purposes here, i.e. only of importance are the terms for temporal derivative and the Hubble-induced velocity space migration, would then attain the following form:

$$\frac{\partial f}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}[v^2 \dot{v}_H f(v,t)]$$

where the term on the right side describes the change of the distribution function $f(t, v)$ under the Hubble-induced velocity drift migration v_H analogous to the velocity space drift formulated in Fahr (2007). In this case here now this drift is connected with the fact that particles which move with a velocity v into the direction $\vec{v}$ within a time increment dt suffer a velocity change $vH = -v \cdot H$ with respect to the new reference place. This consequently then allows to write the above kinetic transport equation in the following form:

$$\frac{\partial f}{\partial t} = -\frac{1}{v^2}\frac{\partial}{\partial v}[v^2(vH)f(v,t)] = -\frac{H}{v^2}\frac{\partial}{\partial v}[v^3 f(t,v)]$$

Introduction of the normalized distribution in the form $f(t, v) = n(t) \cdot \bar{f}(v)$ (i.e. separation of variables!) then allows to write:

$$\frac{1}{n}\frac{\partial n}{\partial t} = \frac{\partial}{\partial t}\ln(n(t)) = -\frac{H}{v^2}\frac{1}{\bar{f}(v)}\frac{\partial}{\partial v}[v^3\bar{f}(v)] = -\frac{H}{v^2}[3v^2 + v^3\frac{\partial}{\partial v}\ln[\bar{f}(v)]]$$

Taking now advantage of the earlier solution for $n(t) = n_0 \exp[-4H(t - t0)]$ then leads us to:

$$\frac{\partial}{\partial t}\ln(n(t)) = -4H = -\frac{H}{v^2}\frac{1}{\bar{f}(v)}\frac{\partial}{\partial v}[v^3\bar{f}(v)]$$

or:

$$4v^2\bar{f} = 3v^2\bar{f} + v^3\frac{d\bar{f}}{dv}$$

or:

$$\frac{1}{\bar{f}(v)}\frac{d\bar{f}}{dv} = \frac{1}{v}$$

or:

$$ln[\bar{f}(v)] = \int \frac{dv}{v}$$

or:

$$\bar{f}(v) = \bar{f}_0 \cdot \exp[\ln v - \ln v_0] = \bar{f}_0 \frac{v}{v_0}$$

One could perhaps hope that this is the solution of the problem, but it is easy to see that this is in fact not the case, since the found solution for $\bar{f}(v)$ is not normalizable to 1! This seems to express the fact that the solution of the kinetic transport equation (Equ, (1)) does not allow for a separation of variables t and v!

Conclusions

With the above result we are now finally lead to the statement that a correctly derived Vlasow equation for the cosmic gas particles in a Hubble universe leads to a collision-free expansion behaviour that neither runs adiabatic for the cosmic gas, nor does it conserve the initially Maxwellian form of the distribution function $f_o(v, t)$. Under these auspices it can, however, also easily be demonstrated (see Fahr and Loch,1991) that under ongoing collisional interaction of cosmic photons with such cosmic particles via Compton collisions in case of non-Maxwellian particle distributions do unavoidably lead to deviations from the Planckian blackbody spectrum. This makes it hard to be convinced by a pure Planck spectrum for the CMB photons at the times around or just after the cosmic matter recombination, i.e. the conclusion should be that what we see in form of the CMB (Cosmic Mikrowave Background) cannot be from the times of recombination of matter. Our results now further raise the question whether or not matter and radiation as ingredients in the GRT energy-momentum tensor have to be carefully reanalyzed on the basis of their unexpected non-equilibrium behaviour. This should perhaps be taken together with most recent results by Fahr and Heyl (2016) and Fahr and Heyl (2017) showing that the energy density of cosmic radiation (i.e. the CMB photons) does not fall off with the fourth, but only with the third power of the scale of the universe.

References

1. Chapman S, Cowling TG (1990) The mathematical theory of non-uniform gases: an account of the kinetic theory of viscosity, thermal conduction and diffusion in gases. Cambridge university press.
2. Cercignani C (1988) The Boltzmann equation. In The Boltzmann equation and its applications (pp. 40-103). Springer, New York, NY.
3. Erkaev NV, Vogl DF, Biernat HK (2000) Solution for jump conditions at fast shocks in an anisotropic magnetized plasma. Journal of plasma physics 64: 561-578.
4. Fahr HJ, Heyl M (2017) How are Cosmic Photons Redshifted?. Advances in Astrophysics 2: 1-8.
5. Fahr HJ, Heyl M (2018) The electro-magnetic energy-momentum tensor in expanding universes, Advances Astrophys 3: 198-203.
6. Fahr HJ (2007) Revisiting the theory of the evolution of pick-up ion distributions: magnetic or adiabatic cooling?. In Annales Geophysicae 25: 2649-2659.
7. Fahr HJ, Fichtner H (2011) Pick-up ion transport under conservation of particle invariants: how important are velocity diffusion and cooling processes?. Astronomy & Astrophysics 533: A92.
8. Pitaevskii LP, Lifshitz EM (2012) Physical Kinetics: Volume 10 (Vol. 10). Butterworth-Heinemann.
9. Partridge R (1995) 3K: The cosmic microwave background radiation, Cambridge Univ.
10. Peacock JA (1999) Cosmological physics. Cambridge university press.
11. Goenner H (1996) Einführung in die spezielle und allgemeine Relativitätstheorie. Spektrum, Akad. Verlag.
12. Serrin J (1959) Mathematical principles of classical fluid mechanics. In Fluid Dynamics I/Strömungsmechanik I 1959: 125-263). Springer, Berlin, Heidelberg.
13. Spitzer, L., Härm, R. (1953) Transport Phenomena in a Completely Ionized Gas. Phys.Review, 89, 977.